

MILITARY EXPENDITURE AND ECONOMIC ACTIVITY: The Colombian Case

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August 11, 2003

¹The authors are General Director of Macroeconomic Policy and Advisor, respectively, of Ministerio de Hacienda y Credito Publico, Republic of Colombia. We wish to thank Maria Ines Agudelo, Carlos Caballero, Alberto Carrasquilla, Roberto Junguito, Juan Ricardo Ortega, Andres Penate, Juan Carlos Pinzon, Carolina Renteria, Fabio Sanchez, Andres Soto, and other members of the Ministry of Defense of Colombia for useful comments and discussions. All errors are our own.

Abstract

We enhance a standard RBC model to account for military expenditure and the costs of an internal conflict or war. The model captures the natural trade-off in military expenditure: crowding out of private consumption and investment but less destruction (and, therefore, higher marginal productivity) of private capital (and labor). Hence, military expenditure below (above) a certain threshold generates a positive (negative) net benefit in terms of output. The model is calibrated to an annual frequency using Colombian data. We find that an increase in military expenditure of 1% GDP (the current policy of Colombian authorities) increases investment and output above the steady state during several periods, before the shock fades away. Eventhough consumption falls on impact (to open up space for the additional military expenditure and private investment), it increases above its stationary trend after three periods, remains on positive grounds thereafter, and the cumulated net gain is positive.

JEL Classification Numbers: E6, E32, H56, N4

Key Words: Real business cycle, stationary state, military expenditure, crowding-out, productivity shock.

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1. INTRODUCTION

Colombia has endured an internal, armed conflict during several decades¹. Yet, since 1999 the country experienced an escalation of the conflict that has been accompanied by a severe slowdown of economic activity. The following tables illustrate this fact.

Table 1²

	<i>Kidnappings</i>	<i>Oil Duct Attacks</i> ³	<i>Blown Electricity Towers</i>
<i>1990-1998</i>	1489	51	20
<i>1999-2002</i>	3269	103	339
$\Delta\%$	120%	102%	1595%

Table 2⁴

	<i>GDP Growth Rate</i>	<i>Unemployment</i> ⁵
<i>1990-1998</i>	3.64%	8.5%
<i>1999-2002</i>	0.40%	15.9%
Δ	-3.24 <i>bps</i>	+7.4 <i>bps</i>

Even though several factors contributing to the post-1999 economic contraction have been identified (capital outflow shock, banking crisis, fiscal imbalances, etc.), the deterioration of the domestic conflict has been at the core of the debate. It has been argued by policymakers that a permanent increase in military expenditure is a necessary condition for the recovery of economic growth. In fact, the government aims to increase military expenditure permanently in at least 1% of GDP.

An interesting question arises: even if additional military expenditure stimulates economic activity, will the benefit (in terms of additional output) outweigh its cost (in terms of spent resources)? This paper tries to answer the latter question by suggesting a real business cycle (RBC) model that incorporates military

¹The main actors of the conflict are, on one side, the FARC-EP (Armed Revolutionary Forces of Colombia-People's Army) and the ELN (National Liberation Army), the two major leftist guerrilla groups, and, on the other side, the legitimate armed forces and the majority of civil society. There are also right-winged, paramilitary, self-defense groups.

²Source: Ministerio de Defensa and Ministerio del Interior, Republic of Colombia.

³Only for the Canolimon-Covenas oilduct given that data for other oilducts is not available for the whole period.

⁴Source: Ministerio de Hacienda y Credito Publico, Republic of Colombia.

⁵Unemployment data covers the whole national territory and goes from 1991 to 2002.

expenditure and the costs of an internal conflict or war. The model captures the natural trade-off in military expenditure: crowding-out of private consumption and investment but less destruction (and, therefore, higher marginal productivity) of private capital and labor. Hence, in the model military expenditure below (above) a certain threshold generates a positive (negative) net benefit in terms of output. Once calibrated to Colombian data, the model reveals that an increase in military expenditure of 1% GDP is expansionary in terms of output, investment and consumption.

Different papers in the literature have studied the link between military expenditure and economic activity. Knight et. al. (1996) make an extension of the standard growth model and conclude that military spending has a growth-retarding effect due to a negative impact on capital formation and resource allocation. Stroup and Heckelman (2001) find that the effect of military expenditure over economic growth in Africa and Latin America is non-monotonic. Indeed, in their model military spending has a positive but diminishing influence over growth if the size of the defense sector is small relative to that of the rest of the economy; this influence turns negative as this sector grows.

Athanassiou et. al. (1998) analyze empirically the impact of defense expenditure over the Greek economy using two different methodologies: an econometric model and a computable general equilibrium model. The results under both methodologies suggest that the defense sector in Greece has had no positive impact over growth. The same result is found by Nikolaidou (1998). Heo (1999) estimates a three-equation econometric model with Korean data and concludes that there are indirect negative effects of military spending over economic growth via exports and investment. A similar model is estimated for Taiwan by Huang (1999). This author finds a positive impact of defense spending over the non-export private sector and a negative one over the more dynamic tradable sector. He also finds that the latter effect offsets the former one.

There are other studies that indirectly address the link between military spending and economic activity by highlighting the economic costs of armed conflicts. For example, Collier (1995) analyzes the economic effects of civil war and internal armed conflicts. He argues that civil war reduces income both in a direct and an indirect way. An armed conflict diminishes income directly through the diversion of resources into military activity, and indirectly through the reduction of the capital stock (physical, human and social). For instance, Imai and Weinstein (2000) show that widespread civil wars are five times more costly than narrowly fought, internal conflicts and reduce economic growth by 1.25 percentage points a year.

This result contrasts with that of Collier (1999) who estimated a 2.2 percentage point loss in the annual growth rate. Using panel data for 147 countries, Hess and Pelz (2002) measure the welfare loss of living in a non-peaceful world and find that the average cost of a conflict is 102.3 dollars per person.

In a paper that studies the economic causes of civil war Collier and Hoeffler (1998) suggest a model where the probability of winning the war depends upon the capacity of the government to defend itself, which is a positive function of military expenditure. Thus, the higher the capacity of the government to finance the defense sector (through a wide taxable base), the higher the chances of dominating or neutralizing rebellion and avoiding the associated economic costs. Indeed, Azam, Collier and Hoeffler (2001) recognize that higher levels of military expenditure may deter rebellion by raising the entry cost, but provide no empirical evidence to sustain this assertion.

However, in a study about the Colombian case, Echeverry et. al. (2001) argue that an increase in military spending during a civil war may have perverse effects for economic growth in the long run, mainly via investment. On the other hand, Trujillo and Badel (1998) find that, between 1991 and 1996, the cost of the armed conflict in Colombia was between 1.5% and 1.1% of the annual GDP level. Pérez (2002) finds that from 1999 to 2001 the costs generated by the destruction of physical capital in Colombia represent 0.64% of GDP. Recently, Contraloría General de la República (2002) estimated an average gross cost of the Colombian armed conflict during the 1991-2001 period of around 1.34% of annual GDP.

In order to carry out the cost-benefit analysis of the Colombian authorities' decision to increase permanently military expenditure in at least 1% of GDP, this paper suggests an RBC model that captures a simple trade-off in military spending: crowding out of private spending (consumption and investment) against lesser destruction (and, therefore, higher marginal productivity) of private capital and labor. As a result, military expenditure below (above) a certain threshold generates a positive (negative) net benefit in terms of output. The model is calibrated to an annual frequency using Colombian data and is used to carry out an experiment consisting of a positive military expenditure shock. The experiment reveals that an increase in military spending of 1% GDP increases investment and output above the steady state during several periods, before the shock fades away. The cumulated net gain after 10 years surmounts to 215 dollars of additional per capita GDP. Eventhough consumption falls on impact (to open up space for the additional military expenditure and private investment), it increases above its stationary trend after three periods and the cumulated net gain is positive (9

dollars of additional per capita consumption after 10 years).

Two caveats apply. First, the shock follows a stationary stochastic process due to the nature of these class of models. Thus, the shock does not capture literally a *permanent* increase in military spending. Still, the high persistence of the shock captures a long-lasting increase in military expenditure. Second, the model measures the net benefit of additional military spending in terms of output (or welfare) levels, not economic growth. The measurement of the net benefit in terms of higher output growth is a topic for a different paper.

The paper is divided in five sections. This introduction is the first one. In the second section the model is presented. Section three discusses the calibration of the model. In section four the results of the military expenditure shock experiment are presented. Section five concludes.

2. MODEL

Consider an economy inhabited by an infinite number of identical, infinitely-lived, risk-averse households that discount the future at rate $1/\beta - 1$. The mass of households is one. In every period each household is endowed with one unit of time to be allocated between labor and leisure. Labor is indivisible like in Hansen (1985). The shift length is fixed at $h < 1$ units of time and each household sends a fraction n of its members to work while the remaining fraction $(1 - n)$ does not work at all. Households have log utility in consumption (c) and leisure:

$$\begin{aligned} U &= \log(c) + n \log(1 - h) + (1 - n) \log(1) \\ &= \log(c) + n \log(1 - h) \end{aligned}$$

Each household owns capital (k) which is used to transfer purchasing power across periods. The capital stock depreciates at rate δ . The final good of this economy, which is the numeraire, is produced with a Cobb-Douglas technology in capital and labor, subject to random productivity shocks $[\exp(z)]$. Indeed, stochastic variable z is governed by a stationary AR(1) process. Final goods can be consumed or accumulated as additional capital by households. Additionally, there is a central government that eats up some of the economy's output in every period.

The central assumption of the model is that, in every period, a fraction γ of the capital stock is lost or destroyed. This is a natural way to capture the

economic impact of a domestic, armed conflict or rebellion. Furthermore, it is assumed that γ is a decreasing function of military expenditure. The idea behind this assumption is that the higher the capacity of the government to finance the defense sector, the higher the chances of dominating or neutralizing rebellion and, thereby, avoiding the associated economic costs [see Collier and Hoeffler (1998)].

Note then that government expenditure (g) can be of two types: military [$\exp(m)$] and non-military [$\exp(s)$]. Both m and s are driven by stationary AR(1) stochastic processes. It is assumed that non-military expenditure simply generates a pure negative income effect on the economy (i.e. the resources are thrown away into the ocean). Military expenditure, on the other hand, does not generate a pure negative income effect: even though it reduces the volume of output available for private expenditure (consumption or investment), it also lowers the fraction of the capital stock that is destroyed in every period. Hence, *the model captures a simple trade-off in military expenditure: crowding out of private consumption and investment but less destruction (and, therefore, higher marginal productivity) of private capital (and labor).*

A central planner solves the following sequential problem:

$$\begin{aligned} \text{Max}_{\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t) + n_t \log(1 - h)] \\ \text{s.t.} \quad & \end{aligned}$$

$$c_t + k_{t+1} + g_t = y_t + (1 - \delta)(1 - \gamma_t)k_t$$

$$y_t = \exp(z_t) [(1 - \gamma_t)k_t]^\alpha (n_t h)^{1-\alpha}$$

$$g_t = \exp(s_t) + \exp(m_t)$$

$$\gamma_t = b m_t, \quad b < 0$$

$$z_{t+1} = \mu_0(1 - \rho_0) + \rho_0 z_t + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$m_{t+1} = \mu_1(1 - \rho_1) + \rho_1 m_t + v_{t+1}, \quad v_t \sim N(0, \sigma_v^2)$$

$$s_{t+1} = \mu_2(1 - \rho_2) + \rho_2 s_t + \eta_{t+1}, \quad \eta_t \sim N(0, \sigma_\eta^2)$$

k_0 given

It can be deduced easily that, in every period t , the state is given by (k_t, z_t, m_t, s_t) while the control is given by (c_t, n_t, k_{t+1}) .

Mathematically, one can identify the trade-off in military expenditure by rewriting the resource constraint in the following way:

$$a_t + \exp(m_t) = (1 - bm_t)^\alpha \tilde{y}_t + (1 - bm_t)(1 - \delta)k_t \quad (1)$$

where $a_t = c_t + k_{t+1} + \exp(s_t)$ represents non-military expenditure and $\tilde{y}_t = \exp(z_t)k_t^\alpha(n_t h)^{1-\alpha}$ represents output gross of capital destruction. The left hand side (lhs) of the constraint captures the cost of increasing military spending while the right hand side (rhs) captures the associated benefit. Indeed, as the lhs shows, a higher m_t takes away or crowds-out resources available for non-military absorption. However, as portrayed in the rhs and recalling that $b < 0$, a higher m_t enhances the volume of available resources by reducing the destruction of capital and, therefore, increasing the marginal productivity of both capital and labor. Graph 1 at the end of the document depicts the net benefit of military expenditure⁶.

Not surprisingly, the graph shows that additional military spending does not always bring about a positive net benefit on the economy. Indeed, the net benefit decreases monotonically with military expenditure so that there exists a threshold beyond which additional military spending imposes a net cost on the economy. Such threshold is given by the intersection of the curve and the x-axis.

Now, the corresponding dynamic programming problem of the central planner is:

⁶To graph this function the steady state values of \tilde{y} and k were used.

$$\begin{aligned}
V(k, z, m, s) &= \max_{k', n} \{ \log[\exp(z)(1 - \gamma)^\alpha k^\alpha (nh)^{1-\alpha} + (1 - \delta)(1 - \gamma)k \\
&\quad - \exp(m) - \exp(s) - k'] + n \log(1 - h) + \beta EV(k', z', m', s') \} \\
&\quad s.t. \\
\gamma &= bm \\
z' &= \mu_0(1 - \rho_0) + \rho_0 z + \varepsilon', \quad \varepsilon \sim N(0, \sigma_\varepsilon^2) \\
m' &= \mu_1(1 - \rho_1) + \rho_1 m + v', \quad v \sim N(0, \sigma_v^2) \\
s' &= \mu_2(1 - \rho_2) + \rho_2 s + \eta', \quad \eta \sim N(0, \sigma_\eta^2)
\end{aligned}$$

From the first order and envelope conditions the following optimality conditions are obtained:

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left[\alpha \exp(z_{t+1})(1 - \gamma_{t+1})^\alpha k_{t+1}^{\alpha-1} (n_{t+1}h)^{1-\alpha} + (1 - \delta)(1 - \gamma_{t+1}) \right] \right] \quad (2)$$

$$\frac{(1 - \alpha) \exp(z_t)(1 - \gamma_t)^\alpha k_t^\alpha (n_t h)^{-\alpha} h}{c_t} = -\log(1 - h) \quad (3)$$

(2) is the Euler Equation governing the optimal consumption/capital accumulation path. As usual, it demands that the marginal cost and benefit of accumulating as capital an additional unit of output (instead of consuming it) be equated. The marginal cost is given by the forgone marginal utility of consumption today [lhs of (2)]. The marginal benefit is given by the expected discounted value of the marginal utility obtained from next period's marginal productivity of the additional unit of capital together with its non-depreciated fraction [rhs of (2)]. Interestingly, the rhs of (2) reveals that the internal conflict distorts the capital accumulation decision. In fact, the internal conflict (i.e. γ_{t+1}) reduces the expected value of the marginal productivity of capital in the future. This reduces the marginal benefit of accumulating capital and, thereby, implies less incentives to invest.

Equation (3) captures the intratemporal labor-supply decision. It is a standard condition that equates the marginal benefit (in utils) of one additional unit of employment (lhs) to the marginal cost (also in utils) of supplying that additional unit of employment (rhs). The lhs of (3) shows that the internal conflict also distorts the labor-supply decision. Specifically, the internal conflict (i.e. γ_t) reduces

the marginal productivity of employment. Hence, the internal conflict reduces the marginal benefit of supplying labor and, thereby, implies less incentives to operate productively in the market.

In sum, the model captures the economic costs of the conflict with the intertemporal and intratemporal distortions embedded in the optimality conditions (2) and (3). By reducing the incentives to invest and to work, both distortions imply a smaller volume of output (and welfare) in the stationary state of the economy, relative to a peaceful world.

3. CALIBRATION

Parameters were calibrated to an annual frequency using Colombian data for the period 1952-1997 (see calibration appendix). Data was taken from DNP (1998), Sánchez (1994), GRECO (1999), Ministerio de Hacienda, Contraloría General de la República (CGR) and DANE. Military expenditure data comes from DNP (1998) and CGR.

Following Cooley and Prescott (1995), parameter values were chosen so that the model, in stationary state, replicates the following 1952-1997 averages observed in Colombia:

Table 3: Replicated Empirical Regularities

\bar{c}/\bar{y}	\bar{i}/\bar{y}	\bar{g}/\bar{y}	$(1 - \bar{\gamma})\bar{k}/\bar{y}^7$	$\bar{\gamma}$	\bar{n}	<i>labor share</i>	<i>capital share</i>
0.74	0.14	0.17	2.63	0.03	0.90	0.60	0.40

The following table illustrates the calibrated parameter values (see calibration appendix):

Table 4: Calibrated Parameter Values

β	h	δ	a	b
0.909	0.615	0.022	0.4	-0.01

The stochastic processes governing z , m and s were estimated using OLS techniques⁸. The next table exhibits the estimated parameter values of such

⁷Recall that observed capital is given by $(1 - \gamma)k$.

⁸In the estimation of the processes of m and s time dummies were introduced to reduce the variance of the estimated residuals. Otherwise, huge, extraordinary, investment expenditures in the military sector introduce a lot of noise into the series.

processes⁹:

Table 5: Estimated Parameter Values

μ_0	μ_1	μ_2	ρ_0	ρ_1	ρ_2	σ_ε	σ_v	σ_η
0	-3	-2.04	0.98	0.94	0.97	0.017	0.078	0.057

4. EXPERIMENT

To estimate the impact of a military expenditure shock over the economy an impulse-response exercise was carried out using the calibrated model. The experiment consisted of a one standard deviation shock to the noise term in the stochastic process driving military expenditure. The shock is equivalent to an increase of 0.32% of GDP in military expenditure. The experiment's results were extrapolated linearly to quantify the impact of a military expenditure shock of 1% of GDP¹⁰¹¹. The cumulated impact over output, consumption and investment per capita over the next ten years is reported in the following table:

Table 6: Cumulated Effects (2003 US\$ and percentage terms)

	y	c	i
1 year	25.2 (1.37%)	-9.9 (-0.82%)	7.5 (4.56%)
2 years	49.9 (2.71%)	-15.7 (-1.29%)	12.4 (7.60%)
5 years	118.8 (6.46%)	-16.8 (-1.39%)	17.8 (10.90%)
10 years	214.9 (11.68%)	8.7 (0.72%)	9.9 (6.08%)

According to table 6, one year after the shock output and investment per capita are 1.37% and 4.56% higher than in steady state. In contrast, consumption per

⁹ μ_0, μ_1 and μ_2 were not estimated; they were fixed for calibration purposes (see calibration appendix).

¹⁰Recall that the Colombian government's policy is to increase military expenditure permanently in 1% of GDP. This expenditure hike was initially financed with a once-and-for-all 1.2% wealth tax on households and firms with net worth above COP \$169.5 million (between US\$58,000 and US\$59,000).

¹¹Even though the shock in the experiment does not capture literally a *permanent* increase in military spending (the process driving m is stationary), the high persistence of the shock captures a long-lasting increase in military expenditure (see Table 5).

capita lies 0.82% below its steady state level. The intuition is quite simple. The military expenditure shock not only increases aggregate demand (crowding-out private consumption and investment), but also operates as a positive productivity shock inducing an increase in investment (and labor supply). The latter effect over investment offsets the former one (i.e. investment goes up). Due to the resource constraint and despite the increase in output (due to the positive productivity shock), the increase in military expenditure and investment must be accompanied by a fall in consumption.

But eventhough consumption falls on impact (to open up space for additional military expenditure and private investment), the cumulated output gains allow consumption to increase above its stationary trend three periods after the shock and to remain on positive grounds thereafter. Indeed, the cumulated net effect over all variables is positive. For instance, ten years after the shock the cumulated net gain for output, investment and consumption per capita amounts to US\$215, US\$9 and US\$10, respectively (see table 6).

5. CONCLUSIONS

This paper suggests a model to capture the natural trade-off in military expenditure: crowding-out of private spending but less destruction (and, therefore, higher marginal productivity) of private capital and labor. The model is calibrated to Colombian data. An experiment with the model reveals that a long-lasting, 1% of GDP increase in military expenditure (current policy in such country) is expansionary in terms of output, investment and consumption *levels*.

Given the response of consumption (negative US\$10 on impact but US\$9 - cumulative - after 10 years), the net effect over welfare, measured as the (risk-aversion adjusted) present discounted value of the consumption flow, is not clear. However, in these type of models quantitative results must no be taken literally. They must be interpreted in relation to the response of other variables. Hence, this paper only proves that after a highly persistent, 1% of GDP, military expenditure shock to the Colombian economy, there is a positive cumulated effect over output (US\$215) that is much higher than that over investment (US\$10) and consumption (US\$9). But, again, this does not necessarily mean that the welfare effect is negligible or negative. The dynamics of the model allow for a considerable welfare-enhancing effect.

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7. CALIBRATION APPENDIX

Parameter values were chosen so that the model, in stationary state, mimics some long-run empirical regularities observed in Colombia. Specifically, the parameters were calibrated to an annual frequency using Colombian data for the period 1952-1997. Data was taken from DNP (1998), Sánchez (1994), GRECO (1999), Ministerio de Hacienda, Contraloría General de la República (CGR) and DANE. Military expenditure data comes from DNP (1998) and CGR.

The calibration process requires that the objects of the model have an appropriate empirical counterpart. As is standard in the RBC literature [see Cooley and Prescott (1995)], in this class of models the stock of capital must have as empirical counterpart not only private capital, but also public capital and the stock of consumer durables. Additionally, net exports must be treated as investment given that the model portrays a closed economy. Thus, measured investment should be adjusted to include public investment, consumption of durables and net exports so that the macro identity in the data is properly mapped into the model. Moreover, the proper empirical counterpart of the model's output (y) should be observed GDP adjusted to include income imputable to public capital and to the stock of consumer durables. However, due to the nature of this paper's model and to the nature and availability of Colombian data, the data treatment must deviate a little from the standard technique suggested by Cooley and Prescott (1995).

In contrast to the U.S., total investment in Colombian national accounts include not only private investment but also public investment. Consequently, the adjustment of observed GDP and observed investment to account for public investment should not be relevant in Colombian data (i.e. the adjustment is already in the observed data). Nonetheless, military expenditure in the model is an abstraction of both operational and investment expenditures in the defense and security sector. As a result, non-military expenditure in the model should also refer to both operational and investment expenditures in the non-military sectors of the government. Therefore, public investment was extracted from the total investment series in the national accounts and added to the public expenditure series. Of course, public capital and income imputable to public capital were also extracted from the observed total capital stock and GDP series, respectively¹². In

¹²To compute income imputable to public capital (k_G) the methodology of Cooley and Prescott (1995) was employed. Income imputable to public capital (y_G) is given by:

$$y_G = (r_G + \delta_G)k_G$$

sum, in the model capital as well as investment are private.

Due to data limitations, durable consumption had to be treated as pure consumption. Hence, the stock of consumer durables could not be added to the capital stock, observed investment could not be augmented with durable consumption and observed GDP could not be adjusted to include income imputable to the stock of durables. It is also important to highlight that, according to the model, the stock of observed or measurable capital is given by $(1 - \gamma)k$ instead of k . Thus, for calibration purposes, the observed average capital-output ratio was divided by the mean of $(1 - \gamma)$ to obtain the correct empirical counterpart of the model's k/y . Finally, it should be noted that employment in the model (n) is to be interpreted as (number of occupied persons)/(economically active population).

The calibration process was divided in three steps.

7.1. Step 1: (b, μ_1)

Let a bar above a variable represent its steady state level. With this notation steady state output is given by:

$$\bar{y} = \exp(\bar{z}) [(1 - \bar{\gamma})\bar{k}]^\alpha (\bar{n}h)^{1-\alpha}$$

This is:

$$\begin{aligned} \bar{y} &= (1 - \bar{\gamma})^\alpha \exp(\bar{z})\bar{k}^\alpha (\bar{n}h)^{1-\alpha} \\ &= (1 - \bar{\gamma})^\alpha \tilde{\bar{y}} \end{aligned}$$

where $\tilde{\bar{y}} = \exp(\bar{z})\bar{k}^\alpha (\bar{n}h)^{1-\alpha}$ represents steady state output gross of capital destruction (i.e. steady state output without conflict). Note:

$$\frac{\bar{y}}{\tilde{\bar{y}}} = (1 - \bar{\gamma})^\alpha < 1$$

It can be deduced that:

$$1 - cc = (1 - \bar{\gamma})^\alpha$$

where cc stands for the cost of the conflict in terms of output levels. Thus:

where r_G is the rate of return of public capital and δ_G is its rate of depreciation. To compute income imputable to public capital $r_G = 10\%$ and $\delta_G = 7.3\%$ were taken from Sánchez (1994).

$$\bar{\gamma} = 1 - (1 - cc)^{1/\alpha}$$

To pin down the value of $\bar{\gamma}$ one can use the previous equation and an estimate of the cost of the armed conflict in Colombia. Trujillo and Badel (1998) and Contraloría General de la República (2002) estimate that the average net cost of the armed conflict is 1.1% and 1.34% of GDP, respectively. With these estimates the resulting values of $\bar{\gamma}$ are 0.027 and 0.033, respectively. The results presented in this paper were obtained with:

$$\bar{\gamma} = 0.027$$

Note also that:

$$\bar{\gamma} = b\bar{m}$$

where:

$$\bar{m} = E(m) = \mu_1$$

is the unconditional mean of m_t . Thus:

$$b = \frac{\bar{\gamma}}{\mu_1} \tag{1A}$$

Now, recall that the steady state value of military expenditure is given by $\exp(\bar{m}) = \exp(\mu_1)$, while that of other public expenditure is given by $\exp(\bar{s}) = \exp(\mu_2)$. The values of μ_1 and μ_2 are not important as long as they are consistent with the value of total public expenditure in the calibrated resource constraint [i.e. as long as $\exp(\mu_1) + \exp(\mu_2) = \bar{g}$ and $\bar{c} + \bar{i} + \bar{g} = \bar{y}$]. Therefore, the value of μ_1 was set arbitrarily at:

$$\mu_1 = -3 \tag{2A}$$

while the value of μ_2 was obtained residually in order to guarantee consistency with the calibrated resource constraint (see Step 3 below).

Note then that given:

$$\bar{\gamma} = -0.027$$

equations (1A) and (2A) determine:

$$b = -0.009$$

7.2. Step 2: $(\alpha, \delta, \beta, h)$

Note that:

$$\alpha = \frac{\left(\frac{\partial \bar{y}}{\partial \bar{k}}\right) \bar{k}}{\bar{y}}$$

Hence:

$$\alpha = \text{capital share} \tag{3A}$$

Now, consider the resource constraint in steady state:

$$\bar{c} + \bar{i} + \bar{g} = \bar{y}$$

Given that:

$$\begin{aligned} \bar{i} &= \bar{k} - (1 - \delta)(1 - \bar{\gamma})\bar{k} \\ &= (\delta + \bar{\gamma} - \delta\bar{\gamma})\bar{k} \end{aligned}$$

Then:

$$\bar{c} + (\delta + \bar{\gamma} - \delta\bar{\gamma})\bar{k} + \bar{g} = \bar{y}$$

This is:

$$\frac{\bar{c}}{\bar{y}} + (\delta + \bar{\gamma} - \delta\bar{\gamma})\frac{\bar{k}}{\bar{y}} + \frac{\bar{g}}{\bar{y}} = 1$$

or:

$$\frac{\bar{c}}{\bar{y}} + [\delta(1 - \bar{\gamma}) + \bar{\gamma}]\frac{\bar{k}}{\bar{y}} + \frac{\bar{g}}{\bar{y}} = 1$$

After rearranging terms:

$$\delta = \frac{\frac{(1 - \bar{c}/\bar{y} - \bar{g}/\bar{y})}{\bar{k}/\bar{y}} - \bar{\gamma}}{(1 - \bar{\gamma})} \tag{4A}$$

Now, from the Euler Equation [i.e. equation (2)] in steady state:

$$1 = \beta \left[\frac{\alpha}{\bar{k}/\bar{y}} + (1 - \delta)(1 - \bar{\gamma}) \right]$$

Consequently:

$$\beta = \frac{1}{\left[\frac{\alpha}{\bar{k}/\bar{y}} + (1 - \delta)(1 - \bar{\gamma}) \right]} \quad (5A)$$

Equation (3) in stationary state is given by:

$$\frac{(1 - \alpha)}{(\bar{c}/\bar{y}) \bar{n}} = -\log(1 - h)$$

Rearranging

$$1 - h = \exp \left(-\frac{(1 - \alpha)}{(\bar{c}/\bar{y}) \bar{n}} \right)$$

Hence:

$$h = 1 - \exp \left(-\frac{(1 - \alpha)}{(\bar{c}/\bar{y}) \bar{n}} \right) \quad (6A)$$

Note then that given¹³:

$$\Theta = \left[\frac{\bar{k}}{\bar{y}}, \frac{\bar{c}}{\bar{y}}, \frac{\bar{g}}{\bar{y}}, \bar{n}, \text{capital share}, \bar{\gamma} \right] = [2.63, 0.74, 0.17, 0.9, 0.4, 0.027]$$

equations (3A)-(6A) determine:

$$\Psi = (\alpha, \delta, \beta, h) = (0.4, 0.022, 0.909, 0.615)$$

¹³The value for $\bar{\gamma}$ was obtained in Step 1 above. The rest of the data (i.e. $\bar{k}/\bar{y}, \bar{c}/\bar{y}, \bar{g}/\bar{y}, \bar{n}, \text{capital share}$) is observed and comes from the sources mentioned at the beginning of this appendix.

7.3. Step 3: $(\mu_0, \mu_2, \rho_1, \rho_2, \rho_3, \sigma_\varepsilon, \sigma_v, \sigma_\eta)$

Parameters $(\mu_0, \rho_1, \rho_2, \rho_3, \sigma_\varepsilon, \sigma_v, \sigma_\eta)$ were estimated using OLS techniques¹⁴:

μ_0	ρ_0	ρ_1	ρ_2	σ_ε	σ_v	σ_η
0	0.98	0.94	0.97	0.017	0.078	0.057

As was mentioned in Step 1, the process to calibrate μ_2 is quite different and requires some algebraic manipulation. From the Euler Equation [i.e. equation (2)] in steady state:

$$1 = \beta \left[\alpha(1 - \bar{\gamma})^\alpha \bar{k}^{\alpha-1} (\bar{n}h)^{1-\alpha} + (1 - \delta)(1 - \bar{\gamma}) \right]$$

This is:

$$\frac{1}{\beta} = \alpha(1 - \bar{\gamma})^\alpha \left(\frac{\bar{k}}{\bar{n}h} \right)^{\alpha-1} + (1 - \delta)(1 - \bar{\gamma})$$

or:

$$\frac{\left(\frac{1}{\beta} - 1 \right) + \delta + \bar{\gamma} - \delta\bar{\gamma}}{\alpha(1 - \bar{\gamma})^\alpha} = \left(\frac{\bar{k}}{\bar{n}h} \right)^{\alpha-1}$$

After rearranging some more the following is obtained:

$$\left(\frac{\bar{k}}{\bar{n}h} \right) = \left[\frac{\alpha(1 - \bar{\gamma})^\alpha}{\left(\frac{1}{\beta} - 1 \right) + \delta + \bar{\gamma} - \delta\bar{\gamma}} \right]^{\frac{1}{1-\alpha}} \quad (7A)$$

Now, equation (3) in stationary state is given by:

$$\frac{(1 - \alpha)(1 - \bar{\gamma})^\alpha \bar{k}^\alpha (\bar{n}h)^{-\alpha} h}{\bar{c}} = -\log(1 - h)$$

This is:

$$\bar{c} = \left[\frac{(1 - \alpha)(1 - \bar{\gamma})^\alpha h}{-\log(1 - h)} \right] \left(\frac{\bar{k}}{\bar{n}h} \right)^\alpha \quad (8A)$$

¹⁴In the estimation of the processes of m and s time dummies were introduced to reduce the variance of the estimated residuals. Otherwise, huge, extraordinary, investment expenditures in the military sector introduce a lot of noise into the series.

Now, consider the resource constraint in steady state:

$$\begin{aligned}\bar{c} + \bar{i} + \bar{g} &= \bar{y} \\ \bar{c} + (\delta + \bar{\gamma} - \delta\bar{\gamma})\bar{k} + \bar{g} &= \bar{y} \\ \bar{c} + (\delta + \bar{\gamma} - \delta\bar{\gamma})\bar{k} &= \bar{y} - \bar{g}\end{aligned}$$

The latter identity can be rewritten as:

$$\bar{c} + (\delta + \bar{\gamma} - \delta\bar{\gamma})\bar{k} = \left(1 - \frac{\bar{g}}{\bar{y}}\right)\bar{y}$$

This is also equivalent to:

$$\bar{c} + (\delta + \bar{\gamma} - \delta\bar{\gamma})\left(\frac{\bar{k}}{\bar{n}h}\right)\bar{n}h = \left(1 - \frac{\bar{g}}{\bar{y}}\right)(1 - \bar{\gamma})^\alpha \bar{k}^\alpha (\bar{n}h)^{1-\alpha}$$

or:

$$\bar{c} + (\delta + \bar{\gamma} - \delta\bar{\gamma})\left(\frac{\bar{k}}{\bar{n}h}\right)\bar{n}h = \left(1 - \frac{\bar{g}}{\bar{y}}\right)(1 - \bar{\gamma})^\alpha \left(\frac{\bar{k}}{\bar{n}h}\right)^\alpha (\bar{n}h)$$

After rearranging:

$$\bar{n}h \left[\left(1 - \frac{\bar{g}}{\bar{y}}\right)(1 - \bar{\gamma})^\alpha \left(\frac{\bar{k}}{\bar{n}h}\right)^\alpha - (\delta + \bar{\gamma} - \delta\bar{\gamma})\left(\frac{\bar{k}}{\bar{n}h}\right) \right] = \bar{c}$$

which implies:

$$\bar{n} = \frac{\bar{c}}{h \left[\left(1 - \frac{\bar{g}}{\bar{y}}\right)(1 - \bar{\gamma})^\alpha \left(\frac{\bar{k}}{\bar{n}h}\right)^\alpha - (\delta + \bar{\gamma} - \delta\bar{\gamma})\left(\frac{\bar{k}}{\bar{n}h}\right) \right]} \quad (9A)$$

Note also that:

$$\bar{g} = \left(\frac{\bar{g}}{\bar{y}}\right)\bar{y}$$

or:

$$\bar{g} = \left(\frac{\bar{g}}{\bar{y}}\right)(1 - \bar{\gamma})^\alpha \bar{k}^\alpha (\bar{n}h)^{1-\alpha}$$

This is:

$$\bar{g} = \left(\frac{\bar{g}}{\bar{y}}\right) (1 - \bar{\gamma})^\alpha \left(\frac{\bar{k}}{\bar{n}h}\right)^\alpha \bar{n}h \quad (10A)$$

Given that:

$$\begin{aligned} \exp(\bar{s}) + \exp(\bar{m}) &= \bar{g} \\ \exp(\mu_2) + \exp(\mu_1) &= \bar{g} \end{aligned}$$

it follows that:

$$\mu_2 = \log[\bar{g} - \exp(\mu_1)] \quad (11A)$$

Note then that given¹⁵:

$$\Xi = \left[\frac{\bar{g}}{\bar{y}}, \bar{\gamma}, \mu_1, \alpha, \delta, \beta, h \right] = [0.17, 0.027, -3, 0.4, 0.022, 0.909, 0.615]$$

equations (7A)-(11A) determine:

$$\mu_2 = -2.04$$

¹⁵The value for $\bar{\gamma}$ was obtained in Step 1 above. The value for μ_1 was also calibrated in Step 1 above. The values for $(\alpha, \delta, \beta, h)$ were calibrated in Step 2 above. The rest of the data (i.e. $\bar{g}/\bar{y}, \bar{n}$) is observed and comes from the sources mentioned at the beginning of this appendix.

GRAPH 1

